

Determine the local extrema of the function

33)  $f(x) = x^4 - 4x^{3/2}$

36)  $f(x) = x^{-2} - 4x^{-1} \quad x > 0$

Determine the relative extrema of the function

$$37) f(x) = \frac{1}{x^2 + 1}$$

$$32) f(x) = x^5 + x^3 + x$$

What you'll Learn About  
 How to find intervals of concavity  
 How to find local extrema using the second derivative

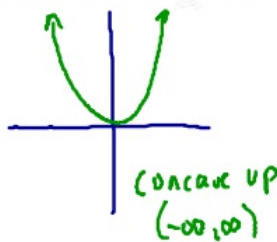
Concavity and Inflection Points

\* concave up  $f'' > 0$   
 concave down  $f'' < 0$

Determine the intervals of concavity and the inflection points

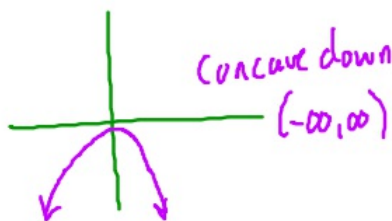
A)  $y = x^2$

$y = x^2$



$y = x^2$   
 $y' = 2x$   
 $y'' = 2 > 0$

B)  $y = -x^2$



$y = -x^2$   
 $y' = -2x$   
 $y'' = -2 < 0$

5)  $f(x) = 10x^3 - x^5$

$f(x) = 10x^3 - x^5$

$f'(x) = 30x^2 - 5x^4$

$f''(x) = 60x - 20x^3$

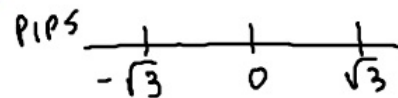
$0 = 60x - 20x^3$

$0 = 20x(3 - x^2)$

$20x = 0$        $3 - x^2 = 0$

$x = 0$            $3 = x^2$

$\pm\sqrt{3} = x$



$(-\infty, -\sqrt{3})$   $f''(-2) = -120 + 160 > 0$   
 $f(x)$  concave up

$(-\sqrt{3}, 0)$   $f''(-1) = -60 + 20 < 0$   
 $f(x)$  concave down

$(0, \sqrt{3})$   $f''(1) = 60 - 20 > 0$   
 $f(x)$  concave up

$(\sqrt{3}, \infty)$   $f''(2) = 120 - 160 < 0$   
 $f(x)$  concave down

Possible Inflection Points (P.I.P.S)

$x = -\sqrt{3}, 0, \sqrt{3}$   
 are all inflection pts. b/c

$f''$  changes sign

b/c  $f''(-\sqrt{3}) = f''(0) = f''(\sqrt{3}) = 0$

P.I.P.S

$f'' = 0$

$f''$  undefined



$(0, 9)$   $f''(1) = -4 < 0$   
 $f(x)$  concave down

$(9, \infty)$   $f''(16) = \frac{1}{2} > 0$   
 $f(x)$  concave up

Determine the intervals of concavity and the inflection points

9)  $f(x) = x(x - 8\sqrt{x}) \quad x \geq 0$

$f(x) = x'(x' - 8x^{1/2}) = x^2 - 8x^{3/2}$

$f'(x) = 2x - 12x^{1/2}$

$f''(x) = 2 - 6x^{-1/2}$

$0 = 2 - \frac{6}{\sqrt{x}}$

$f'' = 0$

$0 = 2 - \frac{6}{\sqrt{x}}$

$\frac{6}{\sqrt{x}} = 2(\sqrt{x})$

$6 = 2\sqrt{x}$

$3 = \sqrt{x}$

$9 = x$  P.I.P.S.

$f''$  und  
 $x = 0$

$x = 9$  Pt of Inflection  
b/c  $f''$  changes  
sign  
and  $f''(9) = 0$

Determine the intervals of concavity and the inflection points

6)  $f(x) = 5x^2 + x^4$